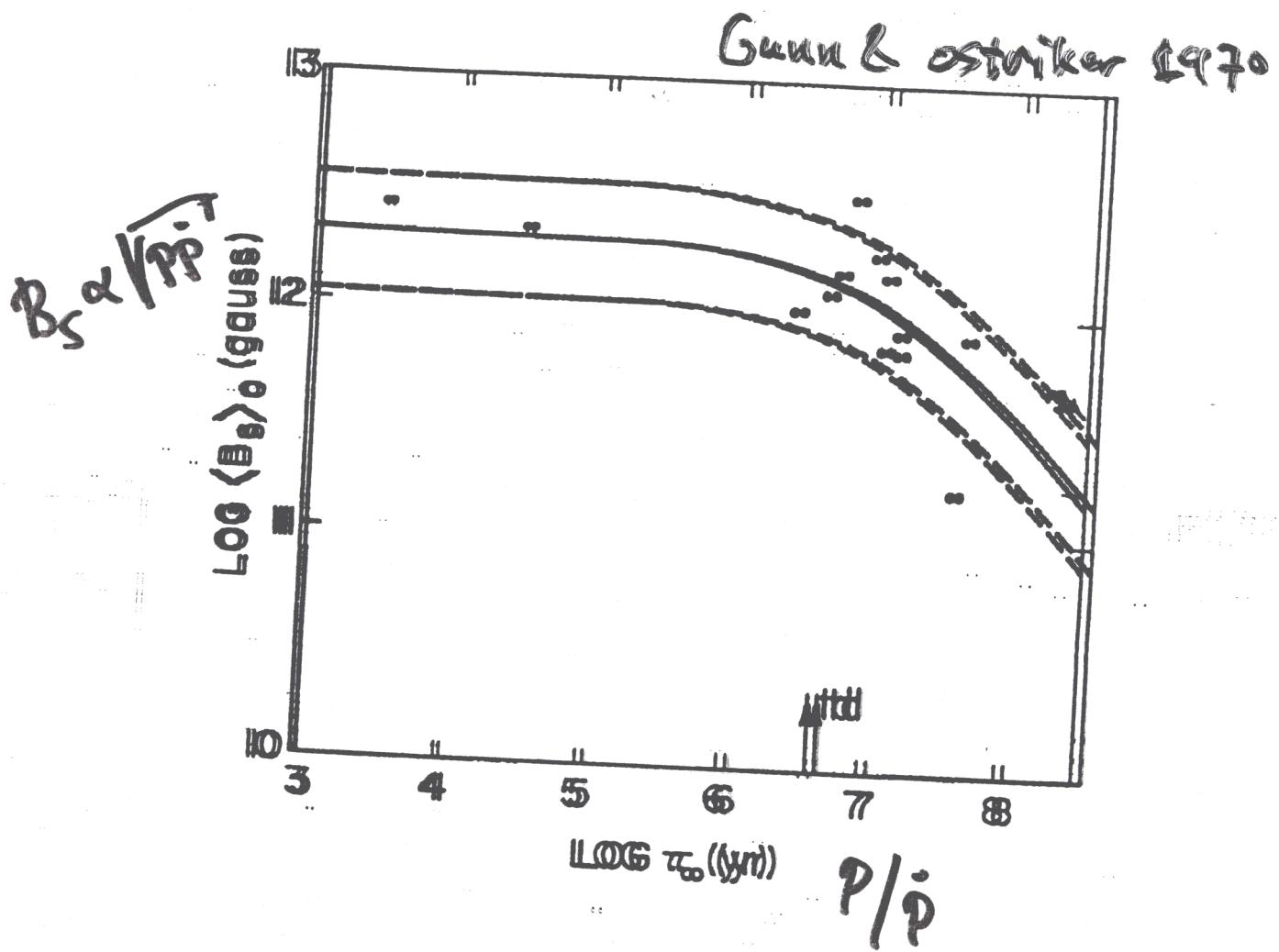


Outline of Talk

- Observational Evidence for field evolution.
- Basics of Ohmic decay and the Hall effect.
- Which effect dominates?
- Ohmic decay evolution
- Hall evolution
- Summary & Speculation.

Field Decay (?)



$$B(t) \approx B_0 \exp\left[-\frac{t}{4 \times 10^6 \text{ yr}}\right]$$

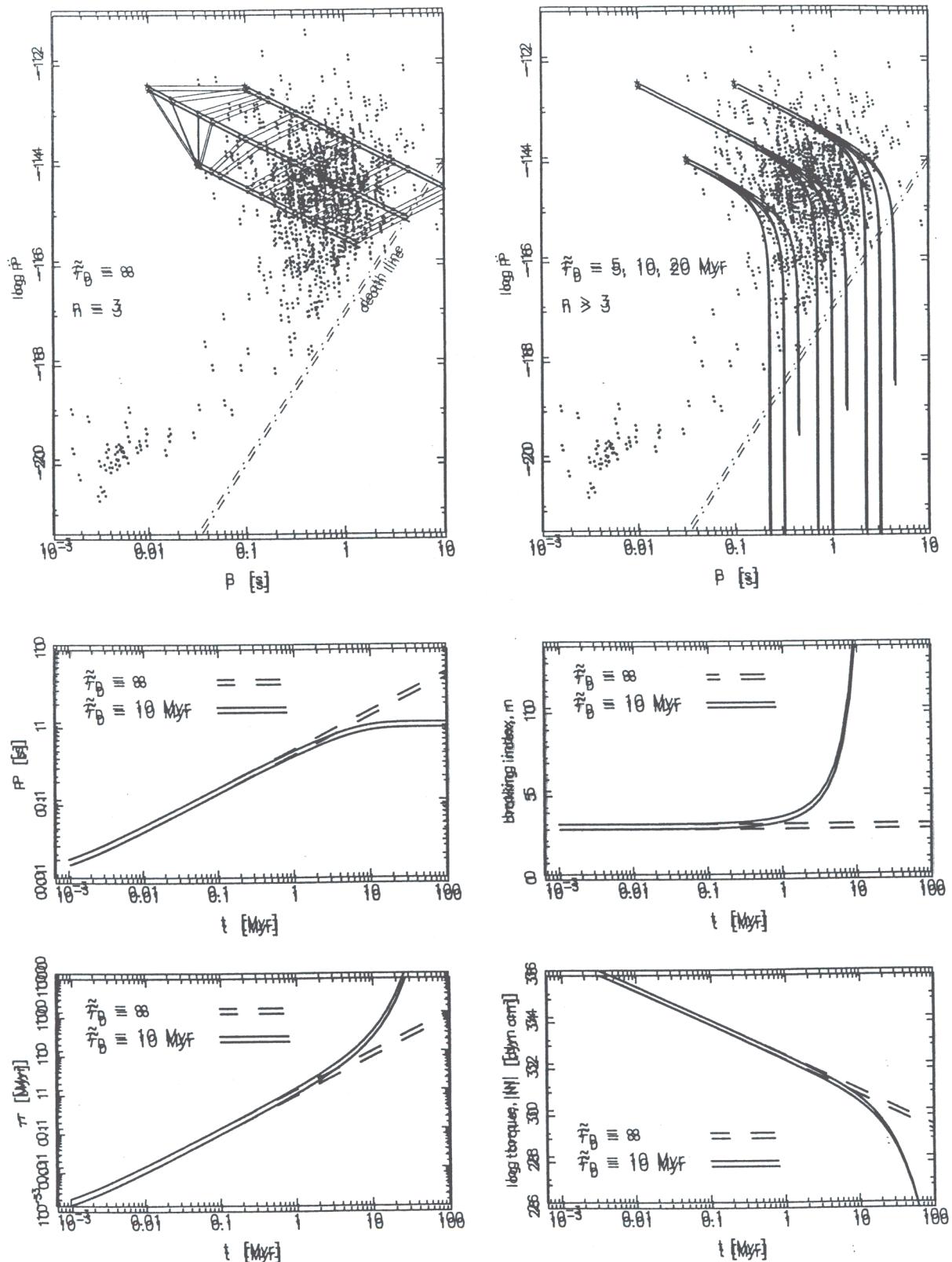
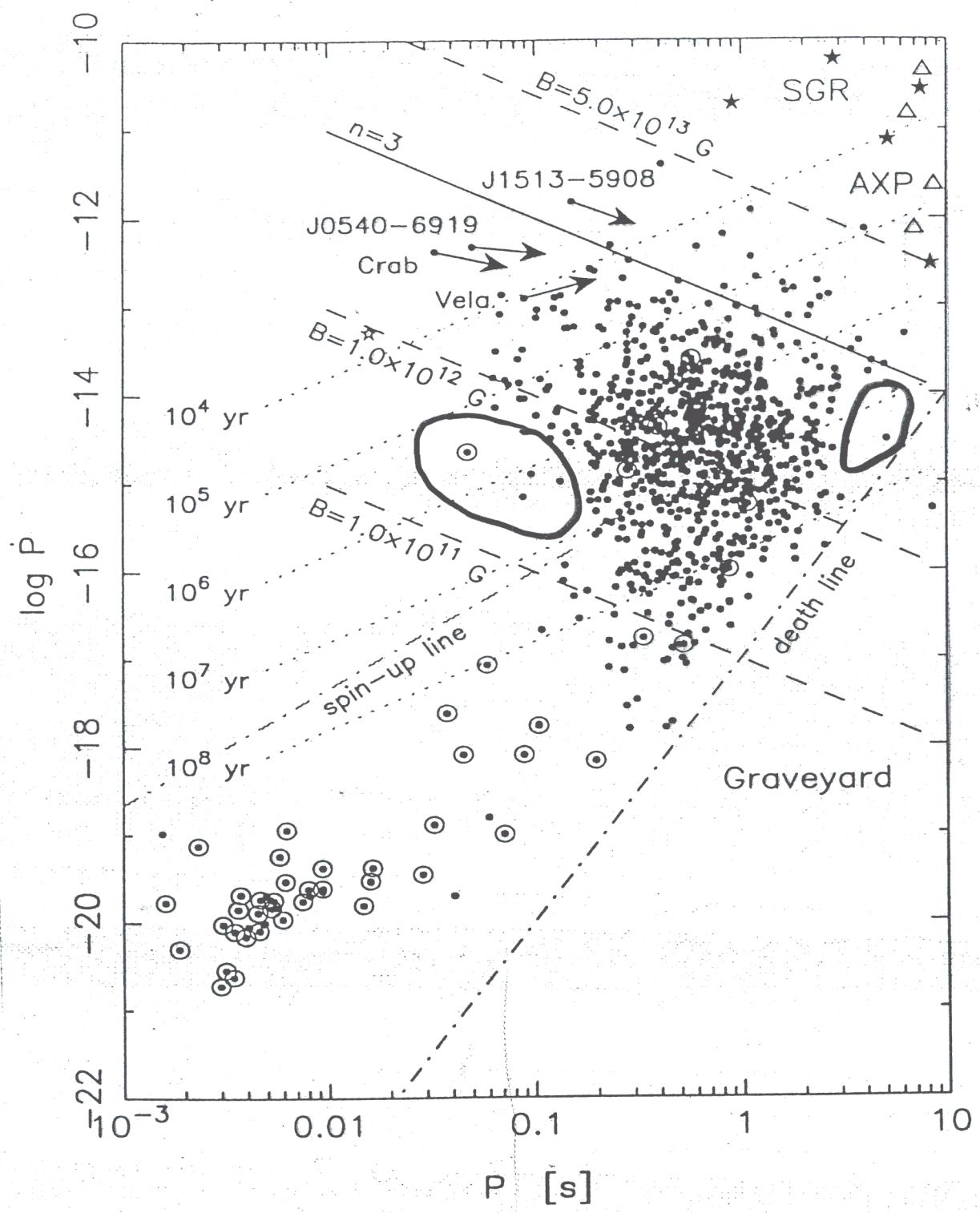


Fig. 2. The top two panels show various evolutionary tracks in the (P, \dot{P}) diagram. The assumed time-scales for enhanced torque decay are written in each panel. The bottom four panels show P, n, τ and N as a function of true age, t = see text.



Tauris & Konar (2001)

• Field Evolution Equations

OHM's Law

$$\underline{\underline{E}} = \frac{\underline{\underline{J}}}{\sigma} + \frac{\underline{\underline{J}} \times \underline{\underline{B}}}{n e c}$$

Ampere

$$\underline{\underline{J}} = \frac{c}{4\pi} \underline{\underline{\nabla}} \times \underline{\underline{B}} = -n e \underline{\underline{v}_e}$$



$$\partial_t \underline{\underline{B}} = -\underline{\underline{\nabla}} \times \left[\frac{c^2}{4\pi\mu} \underline{\underline{\nabla}} \times \underline{\underline{B}} \right] + \underline{\underline{\nabla}} \times (\underline{\underline{n}_e} \times \underline{\underline{B}})$$

Ohmic decay

diffusion

Linear in $\underline{\underline{B}}$

Hall Effect

Advection along $\underline{\underline{v}_e}$

Non linear in $\underline{\underline{B}}$.

Self-contained eqn. for $\underline{\underline{B}}$! B_{ext} , Star must remain in force balance

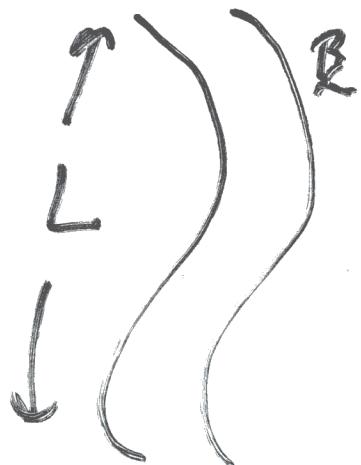
$$\underline{\underline{\nabla}} \cdot (\text{Elastic stress}) + \frac{\underline{\underline{J}} \times \underline{\underline{B}}}{c} = 0 \Rightarrow$$

$$\begin{pmatrix} \text{Strain} \\ \text{Angle} \end{pmatrix} = \theta \sim \frac{B^2}{4\pi\mu} \sim 10^{-3} \left(\frac{B}{10^{14} G} \right)^2$$

Time scales

$$t_{\text{ohm}} \sim \frac{4\pi\sigma L^2}{C^2}$$

$$t_{\text{Hall}} \sim \frac{4\pi NeeL^2}{CB}$$



$$\frac{t_{\text{ohm}}}{t_{\text{Hall}}} = \frac{eR}{m_e^* c} \tilde{\tau}_{\text{coll}} = \underline{\underline{\Omega_L \sigma}} \quad (\sigma = \frac{Nee^2}{m_e^*} \tilde{\tau}_{\text{coll}})$$

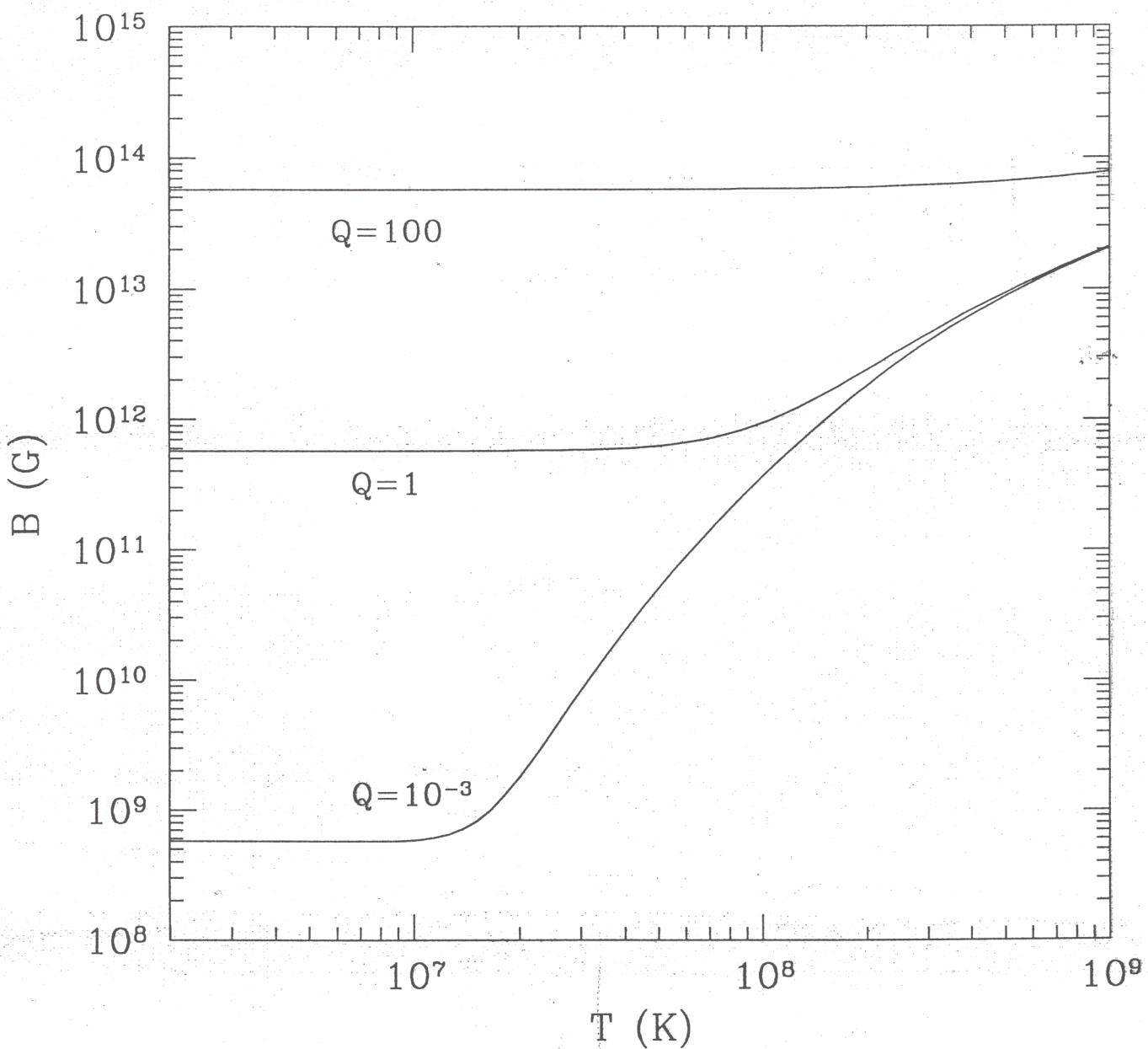
Conductivities at base of crust

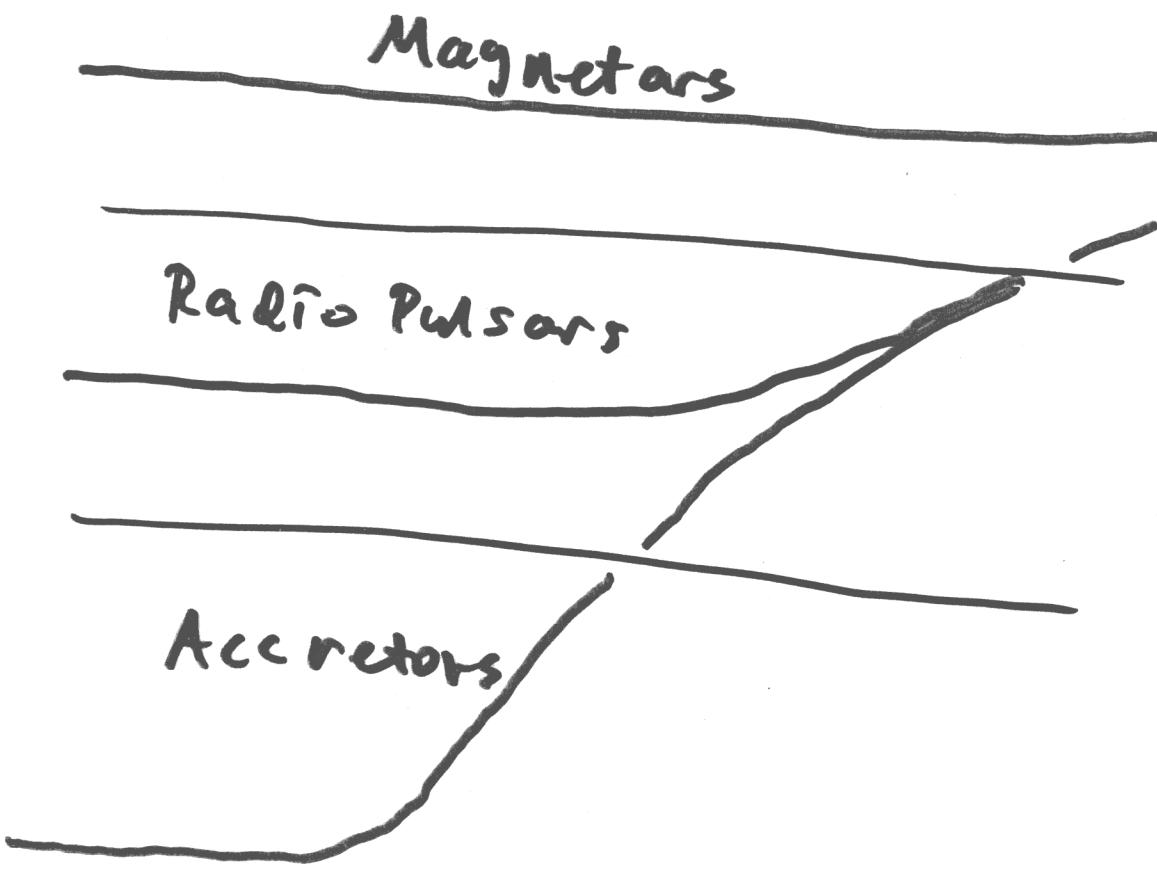
$$\sigma_{e\text{-phonon}} \sim 2 \times 10^{25} T_g^{-2}$$

$$\sigma_{\text{impurity}} \sim 4 \times 10^{25} Q_F^{-1}$$

imperity parameter

When does Hall down at e?





Accreting N.S.

$T \gtrsim 10^8 K$, Q large for rp ashes,

B small

\Rightarrow Ohmic faster than Hall
in accretors.

Isolated N.S.

Passively cooling, $T_8 \approx (t/10^6 \text{ yr})^{-1/6}$ (mod.)
(unc)

$Q \sim 10^{-3}$ (Flowers & Ruderman)

\Rightarrow phonons set conductivity early.

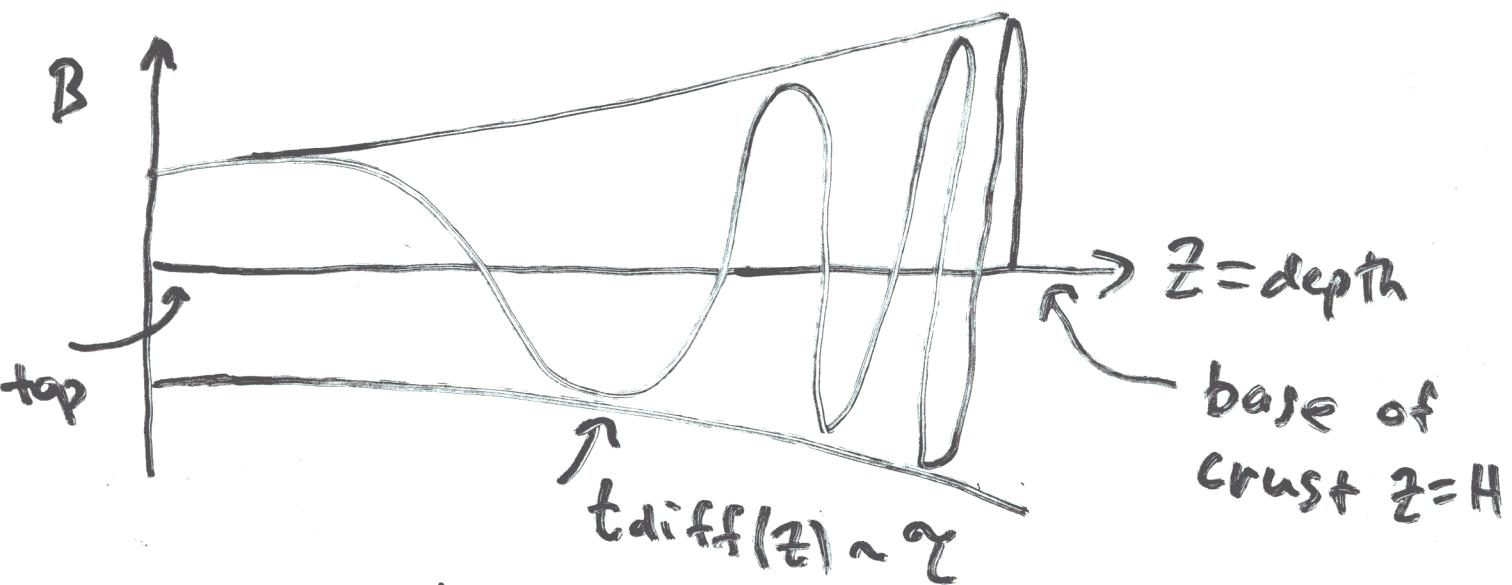
$$\frac{t_{\text{Ohm}}}{t_{\text{Hall}}} \sim \frac{B}{T^2} \sim B t^{1/3}$$

$$\Rightarrow t_{\text{switch}} \sim 10^4 \text{ yr } B^{-3}_{12}$$

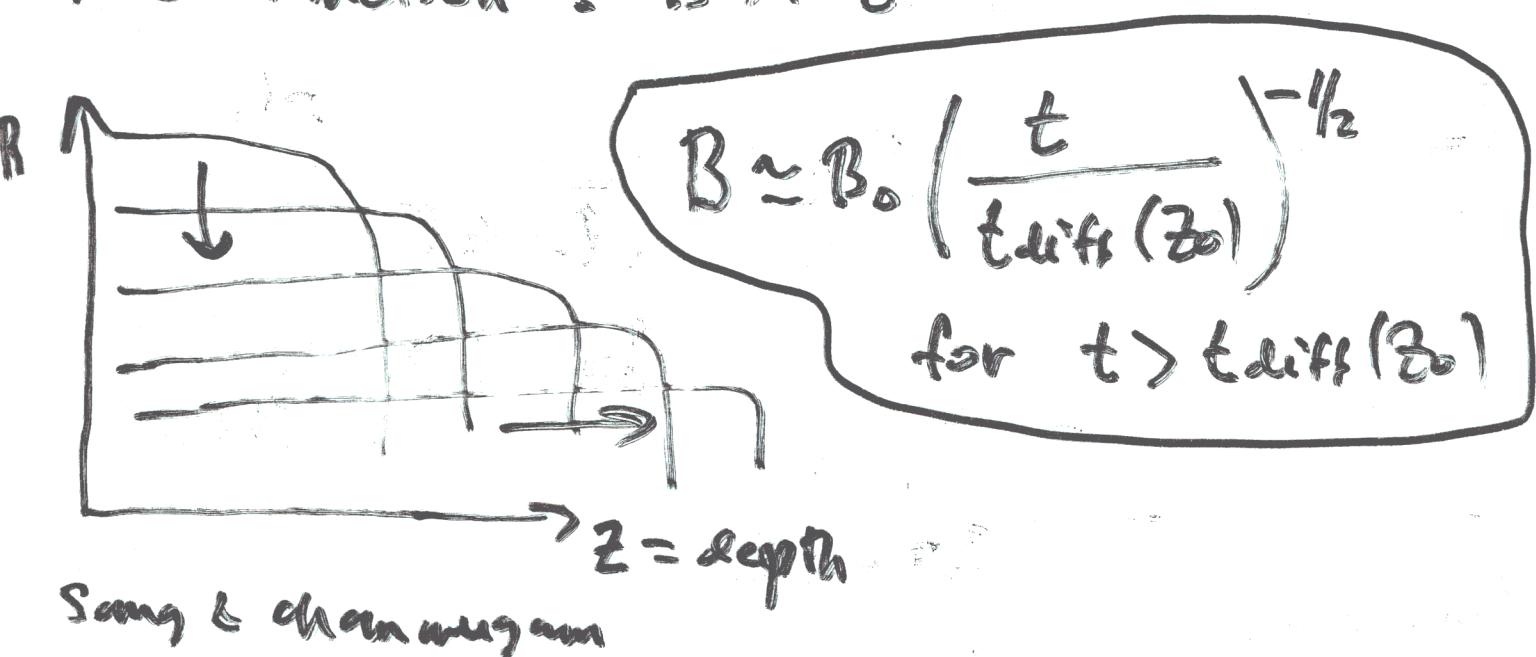
Ohmic dominates Hall before
 t_{switch}

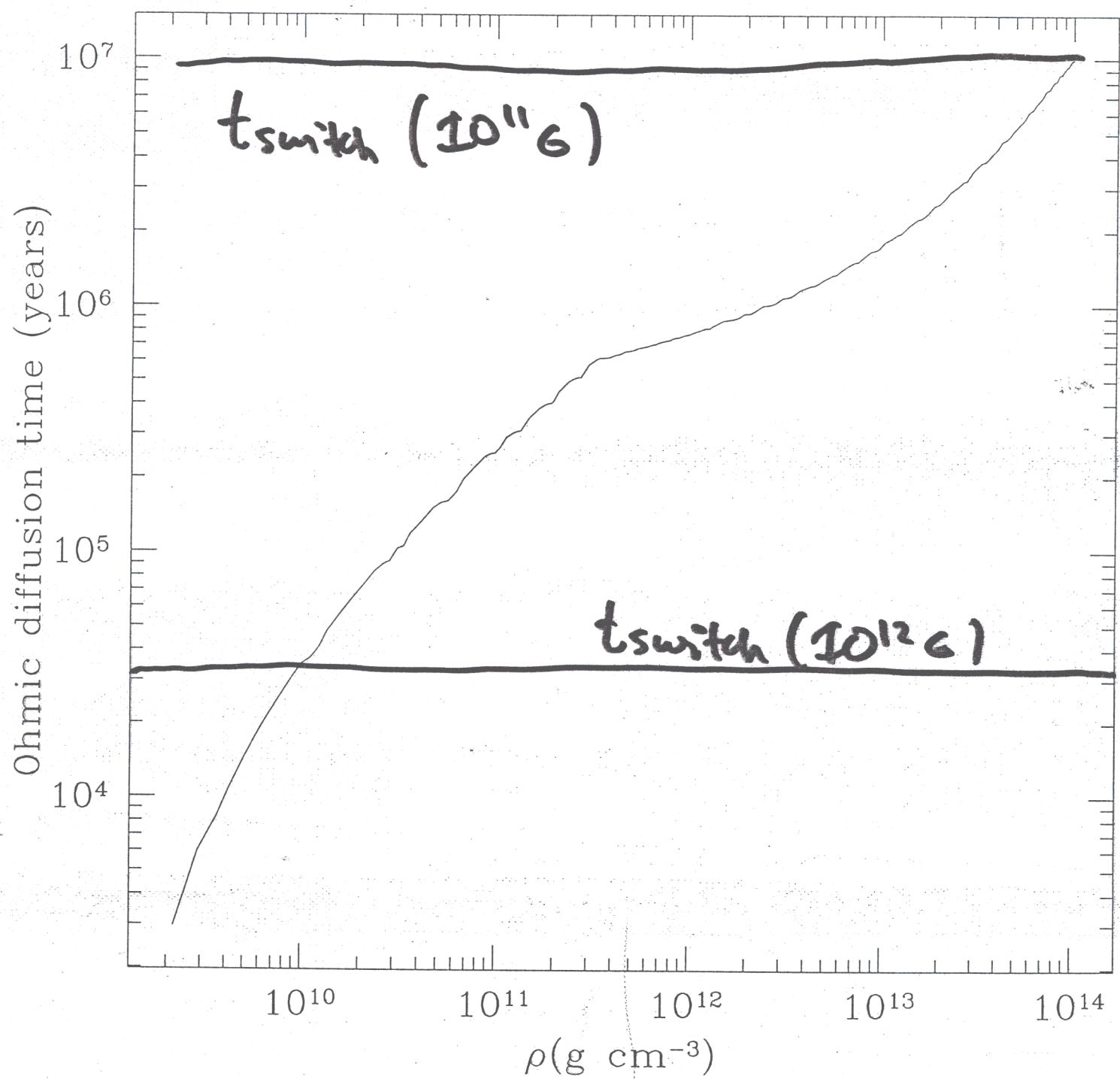
Ohmic Decay

1) Eigen mode: $B \propto e^{-t/2}$



2) δ -function: $B \propto t^{-\beta}$





Hall Effect

How does it change the field?

- 1) Move magnetic energy from one place to another (Jones)

$$(\text{Energy flux}) \sim \frac{c}{4\pi} E_{\text{Hall}} \times B.$$

- 2) Move magnetize energy from one length scale to another (Goldreich & Ruziceneger)



- 3) Non dissipative, but acts as a catalyst for enhanced tidal decay (GR), or crust-cracking (Thompson & Duncan).

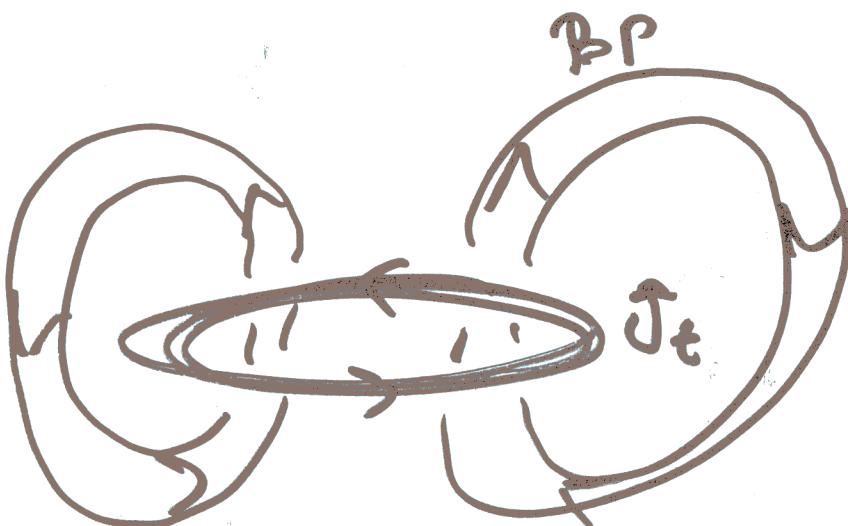
Hall Effect, Ex #: Evolution of Dipole

B_r, B_0 constant, $\Omega_e = V_e/r$

$$\partial_t B_\varphi = \sin\theta \cos\theta r B_r$$

$$\frac{d\Omega_e(r)}{dr}$$

shearing of electron flow



↓ twist



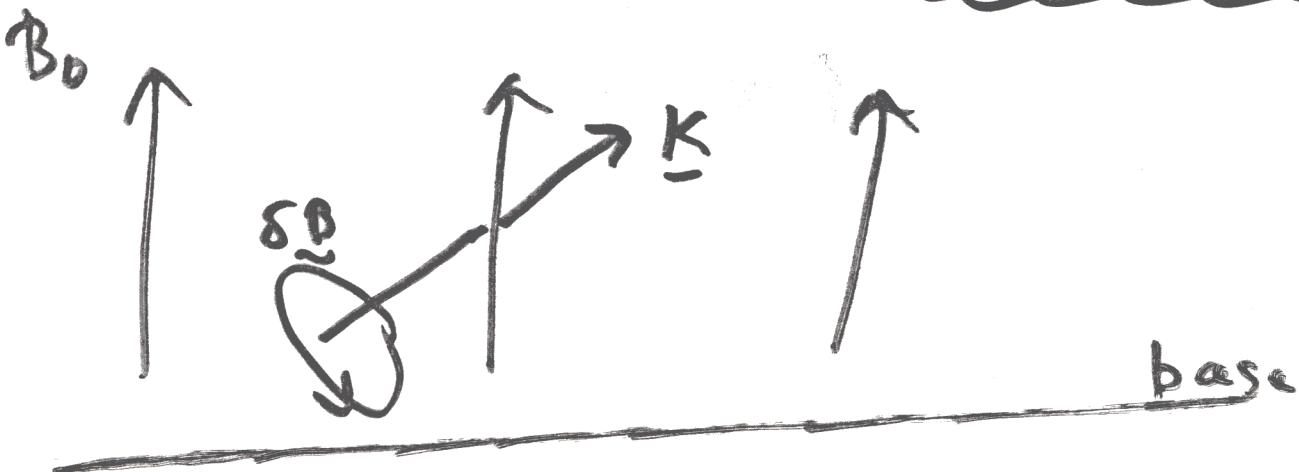
Poloidal dipole



Toroidal quadrupole

Time independent
if no shearing!

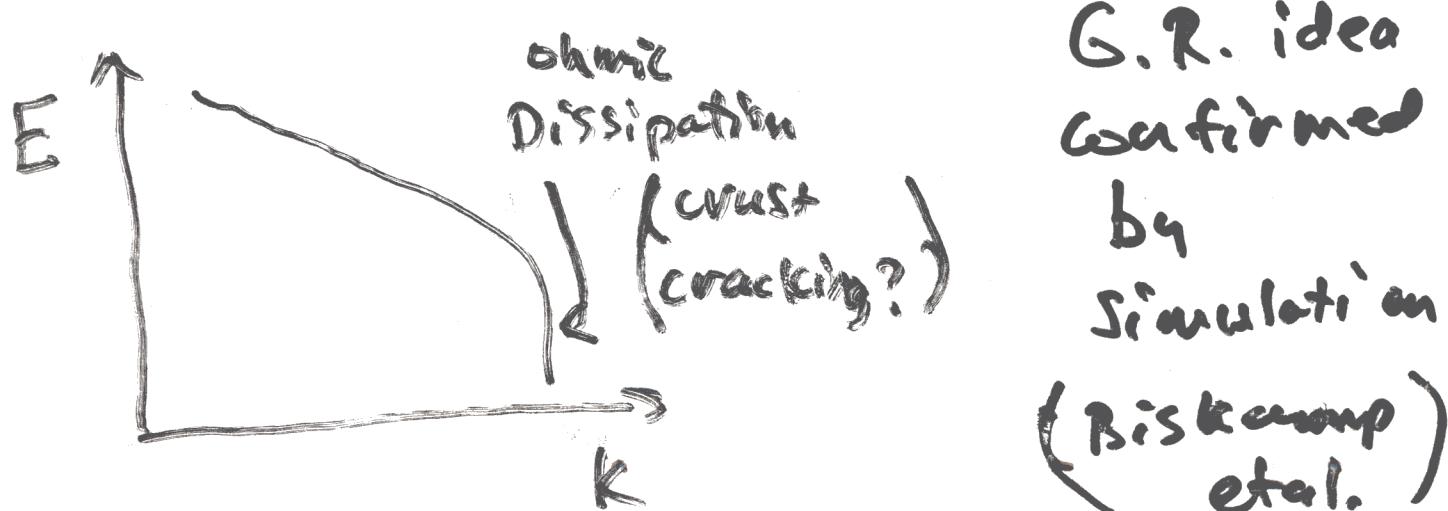
Hall Effect, EX.2: waves & nonlinear interactions



$$\text{Period} \approx \frac{10^7 \text{ yr}}{B_{12} N^2}$$

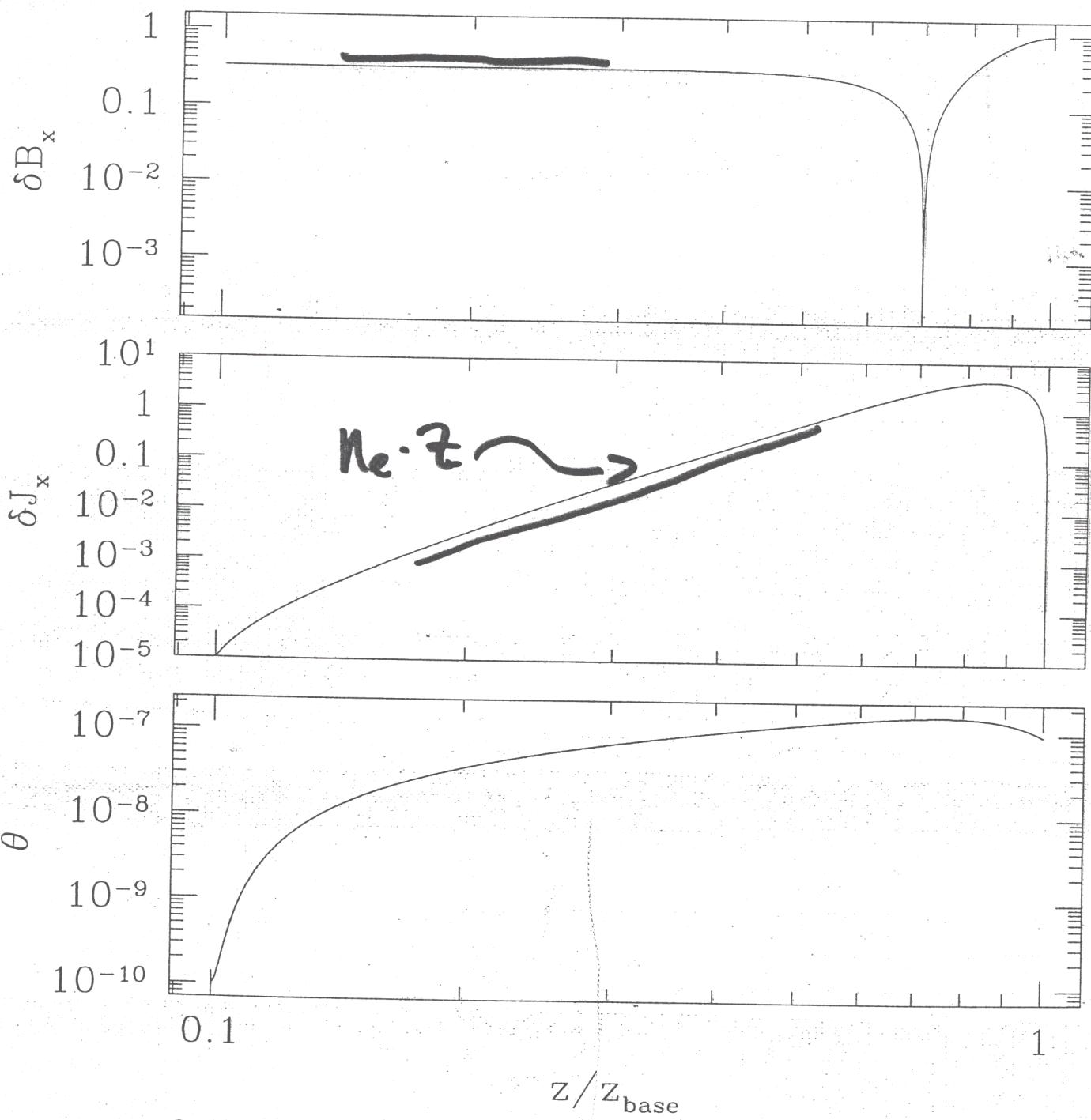
Energy transfer between waves:

$$\frac{\partial}{\partial t} E_1 \sim \int d^3x J_1 \cdot \frac{J_2 \times B_3}{nec} = J \cdot E \text{ work}$$



Hall waves in NS crust.

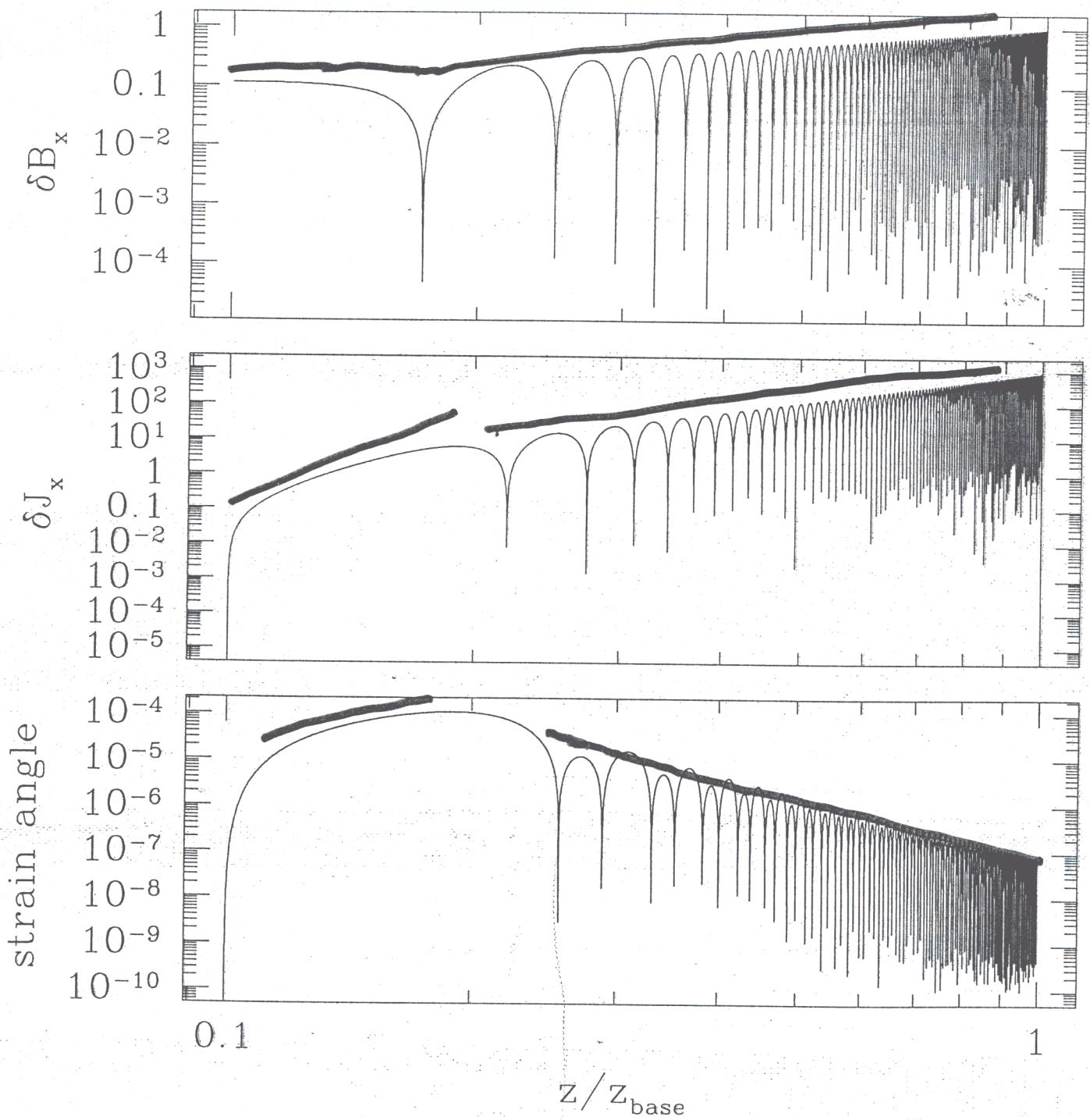
1 node



$$B_{12}=1$$

$$\frac{\delta B}{B}=1 \text{ at base}$$

100 nodes



$$B_{12} = 1$$

$$\frac{\delta B}{B} = 1 \text{ at base}$$

Conclusions

- Early Ohmic decay can significantly change the field in the crust for $B \leq 10^{12}$ G.
- Subsequent evolution is driven by the Hall effect. Large scale field decays on a time scale $\sim 10^7 B_{12}^{-1}$ yr.

Speculation

- Observed surface field may evolve faster if short wavelength $E(k)$ in initial condition!

